



METHODOLOGICAL ASPECTS IN MATHEMATICAL MODELLING OF ECONOMIC PROCESSES IN AGRICULTURE. ECONOMIC-MATHEMATICAL MODELLING OF THE PLANTS CULTIVATION SYSTEM

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Abstract

The complexity of economic phenomena occurring within agricultural holdings, the diversity of the problems arising from the productive activity require modern methods in the decision making process, particularly those of economic-mathematical modelling, which allow in-depth study of the phenomenon. With their help there can be established unknown connections or confirmed hypothesized relationships and dependencies, their use being subject to prior qualitative analysis based on domain-specific models and on the scientific methods explored in the interpretation. We methodologically approached the modelling process and we tried to establish the mathematical model which quite accurately reflects the economic reality, using the notions of linear programming.

Keywords

Linear programming, the optimal system of plants cultivation, sustainable agriculture

1. Introduction

An instrument of rigorous and efficient analysis and resolution of these issues is mathematical programming. Most of the problems in practice are nonlinear programming problems. However nonlinearity involves serious mathematical difficulties, both theoretical and computing ones. In order to be able to use effective methods of linear programming, usually the model „is made linear”, i.e. there are introduced additional assumptions, more or less justified and acceptable from the economic point of view so that we could be led to the linear objective function and constraints. This way it is obtained a 'simplified' model which constitutes a first approximation to the real phenomena. A first step towards a model that better approximates reality is represented by a quadratic programming model. When the stochastic parameters are involved it is recommended to call upon some stochastic modelling methods. Recent research efforts were made to resolve the numerous problems of theoretical and methodological developments in the application of stochastic programming problems in the economy. The difficulty of implementing them makes the analysts turn the problems in individual cases, namely solve them either for a certain level of random variable or for certain achievements, thus becoming their deterministic problems.

2. The mathematical modelling of economic processes in agriculture

Mathematical modelling of processes in agriculture holds a number of peculiarities. The main types of specific phenomena of restrictions on agricultural holdings are as follows:

- restrictions expressing the natural conditions;
- economic restrictions, ensuring a certain level of economic efficiency, as well as the parameters that ensure the compliance of each agricultural unit to market requirements;
- technical and technological restrictions;
- socio-political restrictions;
- structural restrictions which reflect situations of uncertainty created on domestic or foreign markets; risky situations entailed by the influence of bad weather conditions;
- restrictions reflecting local peculiarities related to territorial or climatic characteristics Obviously, in an economic model there must be selected those mathematical restrictions that are considered the most important of all, and once the coefficients have been fixed there must be eliminated those restrictions whose substance is quantitatively redundant.

These are the main functions used in objective models of agricultural planning: maximizing revenue, maximizing profits, minimizing the production cost etc. The profit maximizing is objectively the most often used function if the restrictions on the sustainable component of agriculture is included in the system.

Actual results will depend on the accuracy of the technical-economic coefficients used in developing the mathematical model. The characteristics of production processes in agriculture shall require that, in the determination of economic, technical coefficients, there should be taken account not only the technology but also the human and material resources, the results of the previous period.

3. Economic-mathematical modelling of the plants cultivation system

Like any economic-mathematical model, the model of optimization and location of crop structure has the following elements:

- objective function;
- restrictions;
- variables;
- coefficients și indices.

The economic function. Of all the objective functions the following ones are the most important:

- maximizing profit or net income;
- maximizing net production or overall production;
- minimizing production costs.

The benefit is the most synthetic indicator, more expressive and more conclusive on the economic units efficiency, therefore it is recommended for benefit maximizing to represent the economic function in the analysed models.

The imperative to currently approach the production systems, in line with the policy of saving, preserving and restructuring the energy consumption requires the use of economic criteria in conjunction with energy power, social and environmental protection criteria. Thus it is necessary to take into account a new objective function: energy consumption minimizing. At the same time, the sustainable development reflects economic growth "in actual terms", as measured in one way or another and "negative growth" resulting from the failure to pay for the consumption of the generically called environmental factor.

In mathematical expression the economic function is:

$$(\max) f(x) = \sum_{j=1}^n \sum_{k=1}^m b_{jk} * x_{jk} \quad (1)$$

where:

b_{jk} =the hectare benefit of k crop, on j plot ;

x_{jk} =the k crop area, on j plot.

Restrictions. In the optimization models of the crops structure, the main role of the restrictions is to provide the highest possible economic (positive) value in compliance with certain rules imposed either by technology (the rotation of plants) or by certain

economic requirements (certain amounts of products etc.). The problem is to select of the countless factors the essential ones that may influence the solution, the most important factors in obtaining an optimal solution, those the restrictions should relate to. The value of the solution obtained is determined by the quality of the data included in the model, the more the data number increases the higher the risk to exactly measure the influences of certain factors, certain coefficients expressing these influences. For these reasons it is appropriate to waive restrictions that have smaller influences on the solutions. In the above problem, the restriction may be waived in respect of the area of each crop, leaving only the restrictions relating to the quantities of products. An area restriction is necessary at the unit level, because the sum of all crops must be equal to the unit area. The surfaces of each culture, however, must be the result of optimization. The main restriction to be introduced in the model are:

a) The production restriction that ensures implementation of the products quantities, at least at the planned level. In mathematical expression, this restriction looks as follows:

$$\sum_{j=1}^n q_{jk} * x_{jk} \geq Q_k \quad (2)$$

where:

q_{jk} = the average yield per hectare of k , on every j plot;

Q_k =the necessary amount of products from k crop.

Due to the rapid depopulation of rural areas, certain quantities of market products can be limited if the unit lacks the labor force whose work may exceed certain limits.

b) The constraint surface, ensuring realization of the need for agricultural products, using the existing arable land in the agricultural area. This restriction can be expressed mathematically by:

$$\sum_{k=1}^m x_{jk} \leq S \quad (3)$$

where:

S =the arable land surface (ha).

These restrictions can remove surface restrictions relating to the labor force as they make the model very difficult and can be hardly expressed (e.g., vegetables have a high rate of return, but the unit has the work force only for a certain amount. Thus, the limit will be established both by quantity and area.

c) Crop rotation restrictions. Crop rotation is one component of the plants cultivation which in case of annual structure and crop rotation requires no

additional energy consumption. This group of restrictions ensures the possibility to avoid only one cultivation or the return of a crop on the same lot faster than the biological, sanitary or technical plant protection requirements allow. From the economic point of view the rational crops structure creates the necessary conditions for the prior plant to have a shorter vegetation period, for the soil to keep watered, for the superior soil recovery to be ensured, in compliance with the environmental requirements. The crops structure revaluation in order to save energy requires considerable energy consumption restrictions.

Mathematically this series of constraints can be expressed as follows:

$$\sum_{j=1}^n x_{jk} \leq (\sum_{j=1}^n S_{jk}) / \alpha_k \quad (4)$$

where:

α_k = k crop rotation period, the number of years after which the k crop may lie on the same plot;

S_{jk} = j plot area with k crop.

a) The labor force restrictions are those which provide the possibility to carry out all work at the optimum time. This group of restrictions is inserted only if there were no restrictions limiting certain areas in order to avoid a crops structure that requires more labor force than the unit already has.

The mathematical expression for this restriction is the following one:

$$\sum_{j=1}^n \sum_{k=1}^m \beta_{jk} * x \leq L \quad (5)$$

where

β_{jk} = the labor consumption per hectare of the k crop, on j plot;

L = the highest labor force number that can be ensured by the unit.

Non-negativity restrictions that ensure positive variables. Mathematically, the restriction is given by: $x_{jk} \geq 0$.

The variables. In the optimization models of the crops structure, the variables can represent the areas to be cultivated and the quantities of products that will be obtained. It's worth considering that a model with several thousand variables is not always justified from the economic point of view.

Coefficients and indices. The model coefficients are constant values (in linear programming) that, under these circumstances, make the equations and non-equations system complete. The value of the objective function coefficients, of the resources restrictions is

calculated by different methods or is received from outside the agricultural unit.

Due to the diversity of agricultural production conditions in every agricultural unit, the conversion factors calculation must be made for each individual case, their level generalization being accepted only in very specific cases.

For the elaboration of economic-mathematical models in the optimization of crop structure, there must be calculated a series of technical-economic indicators, and the most important of them are: the average production; overall production; total expenditure; raw material expenses; wages; net production; the benefit; fertilizer/herbicide doses; the number of working days/person or working hours/person; diesel fuel consumption; energy balance; the price of the environment deterioration.

In order to be used in the process of modelling, all these indicators should be calculated on the area unit.

The solution obtained must result in:

- the plots where every plan is going to be cultivated;
- the location of crops areas on plots;
- the average crops per hectare and the whole ones in an agricultural unit;
- the average expenses per hectare and the total ones per unit;
- the profit or the net average income per hectare the total one per unit.

Further to the aggregation of crops areas per year, of yields, of total expenditure and net income, there are revealed the main economic indicators of the solution resulting from the resolution of economic-mathematical model of optimization of the structure and location of the crop.

In solving the economic-mathematical model, one might encounter also cases with no accepted solutions due to some data calculation mistakes or to the formulation of equations and the non-equations, or because of the objectives that cannot be carried out on the existing plots. In this case, it is necessary to return to the model elaboration and data calculation in order to correct the mistakes and solve the problem once again. After getting the solution, it must be analysed and interpreted in two ways:

i) technically and organizationally. This analysis seeks compliance with the imposed technical and organisational restrictions:

- compliance with the established crops rotation, so that no culture should lie on the same plot, but after certain years of rotation;
- every plot must have only one crop to facilitate the agricultural work mechanically performed;
- leave no plots without crops otherwise there should be settled the plants going to be cultivated there;

- the parcels with the same crops should be grouped (together), as far as possible, in order to avoid some displacements of agricultural machinery.

- ii) economically. The second side of the analysis concerns the economic results possibly achieved by applying the obtained solution. To this end there are examined the major economic indicators at the level of the agricultural unit.

The main problems to be analysed are the following:

- the total production of each crop shall be compared with the objectives of the unit. In the system of restrictions on the products quantities, some of which have been limited below (by the sign " \geq " towards the set objective), others have been limited to a certain amount by the sign "=", while others had no plan tasks, with the sign " \leq ". The crops structure and the average yields per hectare shall ensure that the provisions relating to the quantities of products are in accordance with the introduced restrictions. The simplistic solutions are totally inappropriate as they require increasing average productions without considering the proper methods to effectively increase the average yields per hectare;

- the net income per hectare and the total one should be at least at the planned level. Obviously, each unit must carry on a profitable activity. This involves to cover expenses out of income and to get a profit for the economic development of the unit, creating possibilities for the participation of employees in profits etc.

The net income must be analysed on each crop and whole unity.

- the production costs per hectare and the total ones represent another indicator which should be given particular attention in the analysis. The total expenditure should be taken into account in light of the financial resources of the unit.

- the production value is closely linked to the yield per hectare and total level, there will be analyzed the average prices of capitalization and there will be sought ways to get: the higher prices, the continuous quality improvement, products capitalization the earlier possible and in larger amounts per hectare etc.

4. Conclusions

The crop structure is influenced by natural, economic, technological, technical factors etc. An optimal crop structure must simultaneously satisfy multiple requirements: to provide products in the proper quality and quantity to meet demand, to highlight and protect the natural conditions and other factors, to enable the organization of crop rotation, and to provide a reasonable profit (in the present context of sustainable development, the notion of optimal profit changes, the suboptim profit being preferred as it

entails both environmental and social efficiency) to make the enterpriser attain a real economic growth.

In order to obtain high quality agricultural products we should approach the elements of the plants cultivation systems as a unified whole. Starting from this point, the basic feature of the proposed methodology for optimization is the complex approach of the plants crop system, whose functionality is determined by a lot of factors. This methodology allows account to be taken of all the factors influence and the interaction between them, the obtained solutions including both elements which characterize the cultivation as a whole and the ones referring to its components.

In conclusion, the analysis of the obtained solution highlights the way technical and organizational conditions of getting yields and higher incomes are ensured, with lowest costs per product as well as the ability to take certain steps so that the indicators level would meet the management demands of the analysed unit.

Language focus:

Derivatives of certain terms used in the above article:

-yield (v, n)- n.yielder, adj.yielding,adv.yieldingly

-expense(n)- adj.expensive, adv. expensively, n. expensiveness

-index(n), pl. indexes/ indices (especially in technical use)- adj.indexable, n.indexation n.indexer, adj.indexible

-comply (v), often comply with- n.compliance, adj.compliant, adv.compliantly

-cultivate(v)-adj.cultivable, adj.cultivable, n.cultivator, n.cultivation

- environment(n)-adj.environmental, adv.environmentally, n.environmentalist, n.environmentalism

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