



DEVELOPING A MATHEMATICAL MODEL

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Abstract

In this paper we want to realize a mathematical model which is based on statistical data- the research was conducted with the help of university students. The first part describes the interest and people's need for operational research, so that in part - two the phases to describe the underlying model are presented while their classification is realized. In part – three I realize the model itself and I will find the maximum profit the bank gets.

Key words:

Mathematical model,
concept, modeling

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1. Introduction

During the Second World War, the military asked the mathematicians and engineers to study safety issues, issues that have been taken through operational research. Operational research was a major impetus to the end of the first half of this century, having a close relationship with other disciplines such as: computer science, systems analysis, cybernetics, etc.

[1] Adina Rusu, *Operational Researches, Iași 2007*

To understand exactly what operational research does, the emergence and evolution of disciplines that underpin management and organization should be studied and also the links between them. . Among the first concepts that were in the past, the terms of decision or information did not occur. After the First World War were performed analyses that led to research on issues of management and organization of the decision-making and informational aspects, which were not until then.

The main disciplines related to leadership are: cybernetics, research, informatics and general systems theory, namely:

- cybernetics is the science that deals with the regulation and management of complex systems;
- operational research appeared in the period of the Second World War and is the discipline which deals with decision optimization using mathematical modeling;
- informatics is the discipline that deals with data processing using automated processing equipment. The main problems that arise are:

data collection, their preparation, coding, processing and preservation on their equipment;

- You can define the system as any section of the reality in which we see a set of objects, concepts, phenomena connected with the environment through a lot of relationships and act jointly in order to achieve objectives. General systems theory is closely connected to cybernetics and proposes an idea of orientation in management and organization sciences.

II. Basic Concepts used in modeling

The concept of model has been used in modern science and mathematical modeling is as old as were the people's concerns related to knowledge.

Model as a tool of scientific knowledge has been used in many theoretical and practical disciplines.

Shaping is a method of operating the objects, processes or phenomena that are studied using an artificial system, auxiliary or natural and not directly the subject we are interested in, but that with which it is in a certain correspondence objective.

[2] Constantin Anghelache, Mădălina Dumbravă – *Quantitative Methods for Financial-Banking Analysis*

The models are grouped as follows:

- mathematical models
- economic models
 - macroeconomic (refer to the national economy or the economy of a territory)
 - microeconomic (enterprise level, plant, etc.)
- physical models
- biological models

- analogous physical models
- organizational models
- sociological models
- reference models
- verbal-descriptive models
- graphic models
- economic models-describe the behaviour of economic bodies
- cybernetic models- describe the behavior of economic bodies
- cybernetic-economic models study the relationship between inputs and outputs in an economic body
- simulation models set the functioning of micro or macro organisms logical formalization and verification

Mathematical modeling has numerous functions, namely:

- logical formalization and verification
- theorizing
- regulating behavior

[2] Constantin Anghelache, Mădălina Dumbravă – Quantitative Methods for Financial-Banking Analysis

The main phases underlying the achievement of a mathematical model are:

The first phase is the preparatory stage, i.e. the stage of knowledge of the reality in the studied organism.

- a) A second step is to build the model. At this stage of modeling is applied a classical modeling tool, chosen from the wide range of models provided by the theory of operational research.
- b) In the third stage of modeling, the model is facing reality and eventually its experimentation is made.

III. Making a mathematical model

Bank "Piraeus Bank" (Branch Giurgiului) wishes to launch a card for students, for day courses and part-time courses students. Following the launch of the card, the bank has a profit from day courses students from 50 um (ron, dollars, euros, etc.) and from students in part-time courses, the bank has a profit of 40 um (ron dollars, euros, etc.).

In the coming weeks, the branch has 1500 cards, as follows:

- the day classes students are assigned to 300 hours (50 days *6hours/day=36hours)
- the students from part-time courses are allocated 40 hours (5 days * 8 hours / day = 40 hours).

The Bank has in stock 700 cards for day classes students and 800 cards for part-time courses students.

The cards and all their documentation has a total area of 60 m² (square meters) distributed as follows: for the cards from the day classes students an area of 1.6 m² and for the cards from part-time courses students an area of 1.0 m².

The management of the Bank wants to establish the plan to launch the card in the coming weeks in order to maximize the profit.

In order to perform the mathematical model I must synthesize the data in a table, as follows:

Resources	Cards		Available
	Day	Low Frequency	
R1	6 (days)	10 (days)	300 (hours)
R2	1,6 (square metres)	1 (square metres)	60 (square metres)
Profit (u.m.)	100 u.m.	80 u.m.	

To perform the mathematical model I should follow several steps:

1. You need to identify variables and units (the unknowns of the problem are the decision variables-DV).

I mean DV are:

- x1-day (day classes students)
- x2- low frequency (students at part-time courses)

So, VD is accomplished using a mathematical model.

2. You have to calculate the total profit, profit to be maximized using the objective function (OF).

I know the profit for:

- the students from day courses is 100 u.m., and the Bank produces a number of 100*x1
- the students in part-time courses is 80 u.m., and the Bank produces a total of 80*x2

So the objective function is:

$$f(x) = 100 * x_1 + 80 * x_2 = \text{MAX (u.m.)}$$

3. The expression of restrictions that must meet certain conditions: hours, card number, days, etc.

$$\begin{cases} 6 * x_1 + 10 * x_2 \leq 300 \\ 16 * x_1 + 10 * x_2 \leq 600 \\ x_2 \leq 40 \end{cases}$$

4. Terms of e negativity:

$$x_1 \geq 0, x_2 \geq 0$$

According to stages and restrictions we have:

$$\begin{aligned} \max f(x) &= 100x_1 + 80x_2 \\ \left\{ \begin{array}{l} 6x_1 + 10x_2 \leq 300 \text{ (ore)} \\ 16x_1 + 10x_2 \leq 2(m2) \\ x_2 \leq 40 \\ x_1 \geq 0, x_2 \geq 0 \end{array} \right. \end{aligned}$$

Then I solve the problem by a graphical method.

a). Viewing the multitude of admissible solutions that satisfy non-negativity conditions

a1. I view the point that satisfies the inequality:

$$d_1: 6x_1 + 10x_2 \leq 300$$

If $x_1 = 0$ then $10x_2 = 300$, so $x_2 = 30$

If $x_2 = 0$ then $6x_1 = 300$, so $x_1 = 50$

We have the point **A (0, 30)** and **B(50, 0)**

a2. I view the point that satisfies the inequality: $x_2 = 0$

$$d_2: x_2 = 40$$

We have the point **C (0, 20)**

a3. I view the point that satisfies the inequality:

$$16x_1 + 10x_2 \leq 600$$

$$d_3: 16x_1 + 10x_2 = 600$$

If $x_1 = 0$ then $10x_2 = 600$, so $x_2 = 60$

If $x_2 = 0$ then $16x_1 = 600$, so $x_1 = 37.5$

We have the point **D (0, 60)** and **E (37.5, 0)**

b). I want to determine the optimal solution.

I assume you have:

20 cards for day classes

40 cards for part-time courses

I know that:

$$f(x) = 100x_1 + 80x_2$$

So, the point (10, 20) is in the set of admissible solutions from function f.

Then I have:

$$100 * 20 + 80 * 40 = 2000 + 3200 = 5200$$

b1). 5200 if and only if $100x_1 + 80x_2 = 5200$

b1.1). Whether $100x_1 + 80x_2 = 5200$

If $x_1 = 0$ then $80x_2 = 5200$, so $x_2 = 65$

If $x_2 = 0$ then $100x_1 = 5200$, so $x_1 = 52$

b.1.2). I want to make a double profit.

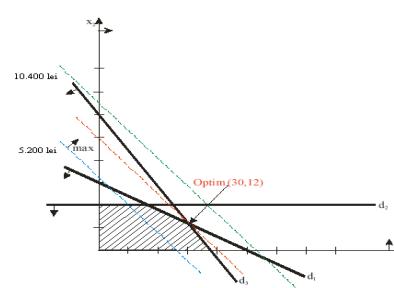
$$100x_1 + 80x_2 = 10400$$

If $x_1 = 0$ then $80x_2 = 10400$, so $x_2 = 130$

If $x_2 = 0$ then $100x_1 = 10400$, so $x_1 = 104$

We have the point **F (0, 130)** and **G (104, 0)**

I have the following graph:



It is noted that the right d_5 does not intersect the shaded area, so I cannot have a profit of 2600. I notice that the lines:

$$100x_1 + 80x_2 = 5200$$

$$100x_1 + 80x_2 = 10400$$

are parallel, so, $f(x) = 100x_1 + 80x_2$ and represent a bundle of parallel lines. If I move the right corresponding to function $f=2500$ in the direction of the peak, then I find that the maximum profit corresponds to the intersection point of the lines d_1 and d_2 .

$$\begin{cases} 6x_1 + 10x_2 = 300 \\ 16x_1 + 10x_2 = 600 \end{cases}$$

The optimal solution of the system is: $x_1^* = 30$ și $x_2^* = 12$.

But, $f(x) = 100x_1 + 80x_2$

Then, $\max(f) = 100*30 + 80*12 = 3000 + 960 = 3960$

The appropriate work programme proposes the maximum profit:

20 day classes card contracts
40 part-time contract cards

And 3960 will be the profit.

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