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EQUILIBRIUM PRICES OF GUARANTEES UNDER UNIT-LINKED LIFE INSURANCE CONTRACTS

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Abstract *The guarantee under a unit-linked contract can be viewed as an option exercisable at the maturity date entitling the policyholder to the greater of the value of the units or the guaranteed amount. The principles of the Option Pricing Model were employed to derive the equilibrium premium for both a single-premium contract and a periodic-premium contract and some numerical simulation were presented. The aim of this study is to determine the equilibrium values of guarantees on single premium contracts and regular premium contracts. Also this research prescribes an optimal investment policy for the insurance company selling these policies*

Key words:
Investment strategies, Investment guarantees, Black-Scholes-Merton Model, Theory of options, Equilibrium prices.

JEL Codes:
G22, G17, G12, C58

1. INTRODUCTION

A unit-linked life insurance policy with an investment guarantee is an insurance policy whose benefit payable on death or at maturity consists of the greater of some guaranteed amount and the value of a reference portfolio which is defined by the deemed investment of a predetermined component of the policy premium in a portfolio of common stocks or mutual fund - the reference fund (Brennan and Schwartz, 1977). One disadvantage of unit-linked contracts as compared with the traditional life insurance contracts is that the policyholder is exposed to the whole of the investment risk, a risk he or she may be reluctant to bear. As a consequence insurance companies have been determined to attach guarantees to these contracts which ensure some minimum level of benefits regardless of the investment results. In other words the insurance company undertakes to bear part of the investment risk in addition to the mortality risk (Boyle and Schwartz, 1977).

Pricing of unit-linked life insurance contracts has generated much interest among researchers and practitioners in the last two decades (Romanyuk, 2006). The payoff in these types of contracts contains both financial and insurance risk elements (Argesanu, 2004), which have to be priced so that the resulting premium is fair to both the seller (insurer) and the buyer (policyholder) of the contract (Romanyuk, 2006).

One of the most challenging issues concerns the pricing of minimum death benefit and maturity benefit guarantees and the establishment of proper reserves for these guarantees. According to Romanian legislation which regulates the unit-linked life insurance market, unit-linked life insurance contracts pass most of the investment risk to the policyholder and involve no investment risk for the insurer. Although the Romanian legislation authorizes the Romanian insurers to offer unit-linked contracts without investment guarantees, this research provides a proposal of a theoretical and empirical basis for pricing and assessing unit-linked insurance contracts with incorporated investment guarantees. Therefore the model proposed by (Boyle and Schwartz, 1977) provides a theoretical framework for valuing these investment guarantees. The prices of investment guarantees are obtained under conditions of market equilibrium using results from the Theory of finance (Theory of options). The maturity guarantee under a unit-linked contract can be viewed as an option exercisable at the maturity date entitling the policyholder to the greater of the value of the units or the guaranteed amount.

The aim of this study is to determine the equilibrium values of guarantees on single premium contracts and regular premium contracts. Also this research prescribes an optimal investment policy for the insurance company selling these policies. This study contributes to the existing literature regarding the issue of appropriate pricing of life insurance contracts and hedging of the risks involved, with an exclusive focus on the unit-linked life insurance contracts with investment guarantees.

The structure of this paper is as follows: Section 2 discusses some previous research on the issue. Section 3 outlines the theoretical framework for determining the equilibrium values of guarantees on single premium contracts and periodic premium contracts. The empirical results regarding the computation of equilibrium prices and the appropriate investment strategy for the insurer issuing these contracts are presented in Section 4. Section 5 provides a summary of the main findings and some concluding remarks.

2. LITERATURE REVIEW

(Table no. 1) presents the main studies oriented to research issues related to life insurance unit-linked products.

*Table 1.***Previews studies**

Previous studies	Research issues related to life insurance unit-linked products
Brennan and Schwartz (1976), Brennan and Schwartz (1979), Boyle and Schwartz (1977)	Described some of the option elements of life insurance products and demonstrated how the option pricing theory of Black and Scholes (1973) could be applied to value these innovative products.
Bacinello and Ortu (1993)	Build on the above papers by considering the case of equity-linked contracts where benefit guarantees are determined endogenously based on the premiums paid.
Ekern and Persson (1996)	Determined premiums for a large variety of equity-linked contracts, including those with payoffs where the contract policyholder chooses the larger of the values of two risky assets (and possibly a guaranteed amount) at maturity of the contract.
Boyle and Hardy (1997)	Analyzed the pricing of and reserving for maturity guarantees for policies where the policyholders' premiums are invested in a specified portfolio which is guaranteed not to fall below a certain level at maturity.
Tiong (2000)	Examined the pricing, hedging and accounting of equity-indexed annuities.
Moeller (2001)	Examined a portfolio of equity-linked life insurance contracts and calculates risk-minimizing strategies in a discrete-time setting for the Cox-Ross-Rubinstein model.
Hardy (2003)	Discussed the modeling and risk management for equity-linked life insurance.
Argesanu (2004)	Focused on the risk analysis and hedging of variable annuities in incomplete markets.
Romanyuk (2006)	Proposed the use of two types of imperfect hedging techniques: quantile and efficient hedging.
Gaillardetz (2006)	Introduced a pricing method for equity-indexed annuities and valuated these products by pricing its death benefits and survival benefits separately.
Boyle and Tian (2008)	Examined the optimal design of equity-linked products with probabilistic constraint.
Mao and Ostaszewski (2008)	Analyzed the pricing of equity-linked life insurance including a minimum interest rate guarantee in a partial equilibrium framework.

Dickson, Hardy and Waters (2009)	Have developed the theory to measure and manage risks that are contingent on demographic experience as well as on financial variables.
Quittard-Pinon and Randrianarivony (2009)	Focused on the pricing of a particular equity-linked life insurance contract where the conditional payoff to the policyholder is the maximum of two risky assets.
Bernard and Boyle (2011)	Proposed the establishment of a natural hedge to reduce the volatility risk of an insurance company's liabilities.
Sweet (2013)	Focused on the pricing and evaluating of the equity-linked annuities.

(Source: Authors' research)

3. METHODOLOGY OF RESEARCH

The economists' interest is to attempt to calculate equilibrium prices for financial contracts of which insurance contracts are a special case. Equilibrium prices are those which may be expected in a perfectly competitive and frictionless financial market. In order to determine equilibrium prices aspects regarding transactions costs, sales costs and institutional constraints are ignored. Calculating equilibrium prices, economists are guided by the arbitrage principle which states that in equilibrium no riskless profits can be made (Boyle and Schwartz, 1977).

Black and Scholes (1971) show that it is possible to form a portfolio of an option and a share so that the rate of return on the combined portfolio is non-stochastic or riskless. In the Black-Merton-Scholes (BMS) framework, one can replicate the payoff of an investment guarantee, which is a essentially a put option, by trading in the underlying stock and the risk-free asset. The Black-Scholes-Merton framework for option valuation is a continuous time model, and is based on more sophisticated market assumptions: the market model should follow a geometric Brownian motion (GBM), there should be no market frictions (no transaction costs and no constraints on trading) (Augustyniak and Boudreault, 2012), short selling is allowed without restriction, and borrowing and lending rates of interest are the same, trading is continuous and interest rates are constant (Hardy, 2003).

Investment strategies: the authors have computed equilibrium values of the guarantees on unit-linked contracts with investment guarantees and in addition this model suggested by Boyle and Schwartz (1977) can be used to implement an optimal investment policy for the insurer selling these contracts. These prices represent the premiums that should be charged under conditions of market equilibrium. This model suggests a dynamic approach to the investment of premiums on unit-linked contracts with incorporated guarantees. This procedure not simply implies just investing the investment components in the reference portfolio and passively leaving them there. Moreover, the optimal strategy dictates that not all the investment component be invested in the reference portfolio. The hedging process can be framed either in terms of the call option or the put option.

4. RESULTS

The dynamic investment strategy prescribed by this theoretical model was computed by simulating the results for a basic single-premium contract and a basic periodic-premium contract. The investment component of the premium is 6971 LEI in the case of the single-premium contract and 349 LEI per year for the periodic-premium contract whose premiums are paid annually. In this research the authors have omitted the transaction costs related to the continuous adjustment and also the mortality component has been ignored in this paper.

The option valuation model provides a theoretical framework for pricing investment guarantees on unit-linked contracts. In order to implement the dynamic hedging strategy, firstly this model ensures an algorithm for calculating the prices that must be charged for these investment guarantees. The model suggested by Boyle and Schwartz (1977) is applied to single premium contracts and periodic premium contracts.

The authors have considered a single premium unit-linked contract offering investment guarantees upon death or at maturity of the contract. If the authors consider a single premium unit-linked contract providing a guaranteed amount g on death or maturity; g may be a function of time or it may be a constant. So when a claim arises the amount payable is the greater of g and the market value of the units purchased in the reference portfolio. Also the date on which the contract will become a claim is known with certainty. The part of the single premium initially invested in the reference portfolio is I (the investment component). This situation corresponds precisely to that of a combination of a call option on a non-dividend paying stock plus the payment of the fixed amount g. It also can be viewed as the combination of the value of

the reference portfolio and a put option to sell the reference portfolio for an amount g at the end of time T. If the value of the reference portfolio is denoted by x, the benefit can be expressed as follows:

$$Benefit=max(g,x)=g+max(x-g,0)=x+max(g-x,0) \quad (1)$$

The value of the put option for a contract maturing with certainty at time T and hence the value of the guarantee is Π_T:

$$\Pi_T = g \times e^{(-r \times T)} \times N(-d_2) - x \times N(-d_1) \quad (2)$$

The values of risk-free asset return r and volatility σ correspond to annually rates under the assumption of continuous compounding.

*Table 2.***Prices of the investment guarantees in single premium unit-linked contracts**

Financial indicators	**Values**					
Investment component (LEI)	*6971*					
Guaranteed amount (LEI)	*6971*					
Risk-free asset return (%)	*6.76*			*13.52*		
Volatility of the returns of the reference portfolio (%)	*9.61*	*19.22*	*38.44*	*9.61*	*19.22*	*38.44*
Time to maturity (T-t)	*Prices of the investment guarantees (LEI)*					
20	0.34	20	0.34	20	0.34	20
19	0.46	19	0.46	19	0.46	19
18	0.61	18	0.61	18	0.61	18
17	0.83	17	0.83	17	0.83	17
16	1.12	16	1.12	16	1.12	16
…						
5	31.20	307.27	1127.48	0.24	52.03	498.44
4	42.13	329.05	1119.93	0.74	72.89	557.22
3	56.50	345.83	1084.25	2.30	101.39	611.12
2	74.40	350.06	1000.13	7.21	138.46	645.22
1	91.94	319.91	815.57	22.60	177.42	614.60

(Source: Authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database. First the authors took into consideration the current rate on 10 year Treasury Bills and then r doubles its initial value. The authors took into consideration the volatility of the returns of the reference portfolio and then σ decreases with 50% and also the volatility doubles its initial value)

(Table no. 2) presents the values of the guarantee for a fixed amount g. According to the empirical results, the value of the guarantee is particularly sensitive to the variance of the reference portfolio, and also the value of the guarantee decreases with increasing maturity. For the higher values of the variance the value of the guarantee increases during the term of the contract before it begins to decrease (for a maturity up to 5 years). However for the various combinations of r and σ the value of the guarantee is zero (for a maturity over 8 years). Most insurers do not issue contracts with maturity less than 7 years.

The periodic premiums contracts are those in which premiums are payable at regular intervals: monthly or annually. The major difference between the regular premium contract and the single premium contract is that in the former case the change in value of the reference portfolio depends not only on the rate of return of the portfolio but also on the rate of addition to the portfolio through investment. Similar with the case of the single premium insurance the value of the guarantee increases dramatically when variance and rate of risk free rate increases. Also these prices increase when maturity of the contract increases (in the early years of the contract the values are lower). An insurance company can reduce the investment risk associated with granting these types of guarantees by selecting those reference portfolios which have the most stable returns.

(Table no. 3) gives the amounts to be invested in the reference portfolio and the risk free asset at yearly intervals for different values of the reference portfolio. This table shows how the relative investment

in the reference portfolio and the risk free asset changes with variations in the value of the reference portfolio at different stages in the life of the contract. The investment is heaviest in the risk free asset when the value of the reference portfolio is low. For high values of the reference portfolio virtually all the financial funds are invested in the reference portfolio. (Table no. 3) shows that the equilibrium value of the maturity guarantee is 71 LEI (Maturity of the contract is certain at the end of the 20 years and the value of the reference portfolio 6971 LEI). The policyholder is assumed to pay 7042 LEI. After the premium is paid, the insurer's liabilities consist of the units in the reference portfolio credited to the policyholder and valued at 6971 LEI, and a put option on the reference portfolio. So the amount invested in the reference portfolio is S= 6813 LEI and the amount invested in the risk free asset is R= 228 LEI.

*Table 3.***Investment in the reference portfolio and risk free asset to ensure hedged position: Single premium unit-linked contract**

Param.		Values									
Risk-free asset return (%)		6.76									
Volatility (%)		19.22									
Increase in value of the reference portfolio (%)		Time to maturity (T-t)									
		20	19	18	17	16	…	4	3	2	1
-80	S	769	714	656	595	531		1	-	-	-
	R	1384	1525	1679	1846	2028		5319	5691	6089	6515
-40	S	3850	3808	3759	3703	3640		1391	937	445	57
	R	526	592	668	754	851		4219	4909	5696	6462
0	S	6813	6793	6771	6745	6716		5680	5444	5136	4690
	R	228	256	287	322	362		1620	1872	2185	2600
60	S	11094	11087	11080	11072	11063		10964	10994	11053	11133
	R	82	91	100	110	121		221	181	111	23
100	S	13907	13904	13900	13897	13893		13893	13912	13932	13942
	R	46	50	55	59	64		55	33	11	-

(Source: Authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database. S represents the amount invested in reference portfolio and R represents the amount invested in risk free asset.)

This algorithm can be extended to include periodic premium policies. This procedure involves the empirical solution of the partial differentiation for a call option. The assumption is that the annual investment component is 349 LEI. In this case the policyholder's account in the reference portfolio increases by 349 LEI each year by virtue of the investment component of each annual premium. If the equilibrium value of the maturity guarantee is 44 LEI (Maturity of the contract is certain at the end of the 20 years and the value of the reference portfolio 349 LEI), the policyholder is assumed to pay 393 LEI. After the premium is paid, the insurer's liabilities consist of the units in the reference portfolio credited to the policyholder and valued at 349 LEI, and a put option on the reference portfolio. So the amount invested in the reference portfolio is S= 393 LEI and the amount invested in the risk free asset is R= 0.31 LEI.

Similar with the previous investment strategy, the investment is heaviest in the risk free asset when the value of the reference portfolio is low. For high values of the reference portfolio virtually all the financial funds are invested in the reference portfolio.

CONCLUSIONS

One of the most interesting life insurance products which have emerged in recent years has been the unit-linked contract (Boyle and Schwartz, 1977). Investment guarantees are very popular features in life insurance policies because in addition to paying a death benefit or a maturity benefit, these policies are tied to the return of

an underlying asset or an actively managed portfolio. Thus, the policy also acts as an investment because the investor's capital is credited with a minimum return. In exchange for this protection, the policyholder pays a higher premium, reflecting the market risk assumed by the insurance company (Augustyniak and Boudreault, 2012).

According to Romanian legislation which regulates the unit-linked life insurance market, unit-linked life insurance contracts pass most of the investment risk to the policyholder and involve no investment risk for the insurer. Although the Romanian legislation authorizes the Romanian insurers to offer unit-linked contracts without investment guarantees, this research provides a proposal of a theoretical and empirical basis for pricing and assessing unit-linked insurance contracts with incorporated investment guarantees. Therefore the model proposed by Boyle and Schwartz (1977) provides a theoretical framework for valuing these investment guarantees. According to the empirical results, the value of the guarantee is particularly sensitive to the variance of the reference portfolio and the maturity of the contract.

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