"Dimitrie Cantemir" Christian University
Knowledge Horizons - Economics
Volume 6, No. 3, pp. 35–40
P-ISSN: 2069-0932, E-ISSN: 2066-1061

www.orizonturi.ucdc.ro

MODELLING THE EFFICIENT FRONTIER OF INVESTMENTS PORTFOLIO

Maria DIMITRIU[1], Maria-Ramona DINU[2], Răzvan Constantin CARACOTA[3]

[1]*Dimitrie Cantemir Christian University, Faculty of Finance, Romania, E-mail:* dimitriumaria1@gmail.com
[2]*Hesper S.A., Romania, E-mail:* maria.ramona.dinu@gmail.com
[3]*Alpha Bank., Romania, E-mail:* razvan.caracota@gmail.com

Abstract *The portfolio is a collection of financial assets (CDs, bills, bonds, common stock) and real assets. The financial securities held in the portfolio are organized according to the investor's interests in categories, maturities, yield levels etc. Combining these financial instruments according to the criteria for investment purposes and the risks involved, is called asset allocation. Optimal portfolio construction is based on analysis and decision period 2007-2012. Since the effects of economic crisis were felt very strongly in the stock exchange activity, in the paper have been defined the following calculation and observation periods: before the crisis: 2007-September 2008, during the crisis: October 2008-March 2010, after the crisis: April 2010-2012.*

Key words:
Efficient markets, efficient portfolio, optimal portfolio selection, assets allocation, risk.

JEL Codes:
G02, G11, G14

1. Introducere

Efficient markets hypothesis was outlined in the early 60s and assumed that securities markets process economic information and react quickly to adjust the prices of financial assets. The main features of the efficient market (market correspondent with pure and perfect competition) are: all investors have free and instantly available information; all participants are rational agents that have homogeneous estimates of future developments.

These theoretical features, purely idealistic, are found in practice in the efficient market under a weaker form:

- Transaction costs are acceptable;
- Free access to the information available is allowed to a large number of investors;
- There is no consistently dominant position for all participants in transactions.

Selection of the optimal portfolio of securities is done by the following hypotheses:

1. Each investment has a certain distribution of rates of return.
2. Each investor is making the investment decision is based on return and risk and they relate their utility curve to the expected return variance and standard deviation.
3. For a given level of risk, investors prefer the variant with the highest rate of return or for a given level of return investors prefer the alternative with the lowest risk.

2. Optimal portfolio construction

Optimal portfolio construction is based on analysis and decision period 2007-2012. Since the economic crisis were felt very strongly in the stock exchange activity in the paper have been defined the following calculation and observation periods: before the crisis: 2007-September 2008, during the crisis: October 2008-March 2010, after the crisis: April 2010-2012.

2.1. General presentation of the selected companies in 2007-2012

To determine efficient investment portfolios there were initially chosen eight companies representing the following areas:

- S.C SINTEZA S.A BIHOR (STZ) - agrochemicals and pesticides, additives for mineral oils;
- SC COMETEX S.A (COXE) - textile and shoe rental and subletting immovable property owned or rented;
- S.C SOCOT S.A TÂRGU MUREŞ (SCTO) - hydraulic structures;
- S.C ŞANTIERUL NAVAL ORŞOVA (SNO) - shipbuilding;
- SC COMCEREAL VASLUI (CCEV) - Growing of cereals (except rice), leguminous crops and oil seeds plant;
- S.C BAT S.A BASCOV (BATV) - services related to oil and gas extraction, excluding surveying;
- S.C HERCULES S.A BĂILE HERCULANE (ERCA) - tourism, spa treatment;

- BANCA TRANSILVANIA (TLV) - monetary intermediation.

Expanding efficient portfolios obtained in the initial phase was achieved by considering new companies as follows:

- S.C BEGA UTILAJE CONSTRUCTII (UTBT) - renting of construction or demolition equipment with related service personnel;
- S.C BEGA MINERALE (BMIA) - quartz sand extraction;
- S.C ZENTIVA (SCD) - manufacture of pharmaceutical preparations;
- S.C ANTIBIOTICE IASI (ATB) - manufacture of basic pharmaceutical products;
- S.C ECOPACK GHIMBAV (AMCP) - manufacture of corrugated paper and paperboard and of containers of paper and paperboard;
- SC TURBOMECANICA (TBM) - manufacture of aircraft and spacecraft.

2.2. Determining the risk and return of securities

In order to determine the risk and return, were monitored the daily prices of each stock traded on BSE and dividends issued by companies. As noted above, the calculations were performed in the first phase, between 2007 and March 2010. The following values were obtained (Table 1):

Table 1. Return and risk 2007- 2012

Company	**Before crisis 2007-09.2008**		**During the crisis 10.2008-03. 2010**		**All period 2007-03.2010**		**After crisis: 04.2010-2012**	
	ER[%]	***σ[%]***	***ER[%]***	***σ[%]***	***ER[%]***	***σ[%]***	***ER[%]***	***σ[%]***
1 – STZ	-0,323	14,22	-0,102	18,96	-0,215	16,69	0,350	8,365
2 – COXE	30,356	82,28	3,369	21,12	17,215	62,25	Not traded since 2010	
3 – SCTO	2,344	38,79	10,070	39,71	6,427	39,34	0,191	10,921
4 – SNO	2,166	17,03	0,462	13,83	1,391	15,95	-0,236	4,039
5 – CCEV	-0,450	11,77	4,684	18,36	2,047	15,55	0,734	9,486
6 – BATV	42,909	140,73	6,825	14,34	25,339	102,89	0,395	13,149
7 – ERCA	7,917	42,59	3,694	15,35	5,862	32,41	1,630	32,746
8 – TLV	-5,445	12,97	17,901	46,46	5,626	35,69	-0,0593	3,477

When analyzing the effects of the crisis on individual asset returns the conclusions are:

- The following companies have developed better in terms of returns: SCTO had an increase of 7,726% ; CCEV had an increase of 5,134%; TLV had an increase of 23,346% ; STZ had an increase of 0,221%.
- Between April 2010 and 2012 the company Cometex was suspended and the return of the other titles have decreased greatly, except the company Sinteza that appreciated by 162,79% and the related risk decreased by 50,12%.

2.3. Determining variants of efficient portfolios

This phase is characterised by determining efficient portfolios, the efficient frontier and the portfolio with minimum risk.

The structure of efficient portfolios is as follows:

All period: 2007-03.2012

Table 2. Efficient Portfolios -1

Variants of efficient portfolios	Efficient frontier	Portfolio with minimum risk		Weights [%]
	ER[%]	***ER[%]***	***σ[%]***	
7 stocks				
1 2 3 4 6 7 8	2,4-7,8	2,4839	14,745	31,86/1,753/9,245/38,817/0,35/7,266/10,709
5 stocks				
1 2 4 6 8	1,5-7,6	2,0645	15,888	36,524/2,55/48,196/1,61/11,12
2 5 6 7 8	3,6-14,4	4,4785	16,106	33,52/63,639/3,66/7,157/22,191
4 stocks				
2 5 6 7	3,0-16,3	4,4052	18,869	5,849/75,374/3,87/14,906
2 6 7 8	5,9-18,5	8,6290	30,903	11,823/7,77/42,12/38,286
2 3 6 8	7,3-15,2	8,4597	33,512	14,838/33,692/4,273/47,197
2 3 6 7	7,5-14,9	8,4610	33,637	15,068/29,86/3,678/51,395
3 6 7 8	6,0-11,1	6,6089	28,444	23,828/3,578/36,66/35,934
2 5 7 8	5,7-10,7	5,7000	21,805	19,112/60,455/0,12/20,312
5 6 7 8	3,0-11,6	3,9714	16,426	65,747/3,448/7,868/22,937

3 stocks				
2 6 7	7,0-20,3	9,7177	39,583	16,506/7,387/76,107
2 3 6	9,3-20,1	10,722	47,226	31,269/63,878/4,853
2 6 8	6,5-20,3	10,387	41,271	21,314/11,603/67,084
6 7 8	5,9-21,6	7,3266	32,603	8,039/48,939/43,022
3 6 7	6,1-13,6	7,1411	35,822	34,164/5,575/60,261
5 6 7	2,6-13,7	3,4915	19,406	79,905/3,479/16,615

Interpretations:

The highest returns have been obtained for the variant with 3 stocks: **236** (COXE, SCTO, BATV): 10,72% with an associated risk of 47,226%, **268** (COXE, BATV, TLV): 10,387% and the risk, 41,271%. One can notice that for an increase of 0,333% in return there is an increase of 5,955% in risk.

For a variant with less risk there is a way of investing in a portfolio of four stocks.

It can be seen that the largest returns corresponds to the lowest risk and the lowest profitability, the highest risk. Thus, one can choose the investment variant 2678 (COXE, BATV, ERCA, TLV).

If we analyze variants of maximum returns, of three and four stocks, it appears that the spread between the returns is 2,206%, while the risk spread is 16,323%, which results in a choice of four stocks variant.

The latter contains two other assets 2 (COXE) and 6 (BATV).

From the results shown in the table above we may conclude the following:

- all considered efficient portfolios include the company Bat Bascov S.A. operating in the oil, as it had a return of 25,339% throughout the period under review;
- as the number of assets in the portfolio increases, the risk decreases;
- there is no efficient portfolio containing all companies analyzed in this first phase.

Crisis period: 10.2008-03.2010

Table 3. Efficient Portfolios -2

Variants of efficient portfolios	Efficient frontier	Portfolio with minimum risk		Weights [%]
	ER[%]	***ER[%]***	***σ[%]***	
5 stocks				
1 2 4 6 8	1,0-7,0	3,1935	12,897	25,054/8,675/42,274/20,382/3,615
2 5 6 7 8	4,0-7,3	5,2258	11,800	14,972/38,833/2,4/35,904/7,891
4 stocks				
2 5 6 7	4,0-6,0	4,8629	11,391	1,632/25,748/65,677/6,943
2 6 7 8	5,0-8,8	5,0000	13,713	0,7314/66,789/11,243/21,236
2 3 6 8	5,9-9,2	6,2097	16,904	26,601/2,339/69,001/2,059
5 6 7 8	5,0-9,2	5,8000	12,873	41,606/6,079/47,36/4,955
3 4 7 8	2,0-10,0	3,3468	13,551	0,62/57,293/41,254/0,834
1 4 6 8	1,0-7,8	3,1937	13,437	5,436/1,386/80,182/12,997
3 stocks				
2 6 7	3,7-6,0	4,9355	13,808	29,233/22,393/48,374
2 3 6	6,0-7,2	6,0000	16,977	22,632/4,84/72,528
2 6 8	6,0-9,2	6,3306	17,138	21,543/71,962/6,495
6 7 8	5,4-9,8	5,8669	15,423	51,258/48,03/0,7112
3 5 6	6,0-7,7	6,2258	15,144	5,774/36,739/57,488
1 2 6	0,8-6,1	4,3871	15,156	26,01/20,93/53,06
2 3 4	0,9-6,4	2,2258	16,509	14,219/0,2608/85,52
3 7 8	4,0-15,5	5,5645	20,132	23,769/3,8/72,431

Interpretations:

There is a uniformity of characteristics of portfolios evaluated. Regardless of the number of shares included, values fall on average between 3-6% for return and 12-16% for risk.

From the results shown in the table above it results the following:

- As the number of assets in the portfolio increases, the risk decreases slightly;
- Companies with the highest frequency of occurrence are 2 – Cometex, 6 - Bat Bascov;
- There is a wide range of investment opportunities.

Before crisis: 2007-09.2008

Table 4. Efficient portfolios

Variants of efficient portfolios	Efficient frontier	Portfolio with minimum risk		Weights [%]
	ER[%]	***ER[%]***	***σ[%]***	
6 stocks				
1 2 5 6 7 8	3,0-7,1	2,6000	15,608	1,635/12,022/70,566/7,264/8,348
5 acţiuni				
2 5 6 7 8	0,0-7,2	0,0000	12,197	12,257/71,715/7,355/8,37/0,3028
2 4 5 6 7	3,0-18,6	3,2000	15,849	5,628/13,297/76,819/3,466/0,79
4 stocks				
2 5 6 7	1,2-25,6	2,0565	16,099	3,594/92,198/2,993/1,215
2 6 7 8	0,0-22,8	0,0000	18,222	23,705/13,932/31,596/30,767
2 4 6 7	6,7-26,0	6,7000	23,313	44,338/1,48/23,489/30,693
4 5 6 7	2,0-17,3	2,0000	16,294	11,378/0,371/28,774/595
3 stocks				
2 6 7	8,5-34,7	15,645	51,629	64,842/34,96/0,19792
2 3 6	9,0-34,35	9,3548	52,196	24,084/75,265/0,6511
1 2 6	1,0-34,3	3,2258	21,648	90,402/4,787/4,811
3 6 7	6,0-15,6	6,0000	41,945	0,736/22,074/77,19
4 6 7	3,0-19,2	3,5484	23,925	96,35/1,784/1,866
5 6 7	1,0-19,0	1,0000	16,285	0,90/31,888/0,67214

Interpretations:
From table 4 it can be seen that the highest return was obtained for three stocks variant 267 (COXE, BATV, ERCA):15,645% and 51,629% risk. A good position has 236 (COXE, SCTO, BATV), with a return of 9,3548% and an associated risk of 52,196%; it is riskier than the first with 0,567% and less effective with 6,2902%, which does not make it attractive to an investor.
If we analyze variants we can see that the spread between the returns is 0,7000 %, while the risk spread is 18,632 %, which results in a choice of four stocks variant. The above period assessed does not provide many investment opportunities.
From the calculations it is observed that the problems arising from the economic crisis affected all analyzed companies, two of them coming out of the market after the suspension. The first is Cometex, and the second, SOCOT Targu Mures was suspended in June 2012 on the grounds of clarification of the division of society.
By plotting the efficient frontier accurately determine the minimum and maximum variation of the pair (return, risk), the point of minimum risk and position to the market.
Portfolio's characteristics are:

$$ER_{min} = 0,097177$$
$$\sigma_{min} = 0,39583.$$

3. Determination of efficient financial investment portfolios based on Sharpe's method
We determine the coefficient values using the BET index (indicator defining market trends) in the period 2007-March 2010.
The results are summarized in Table 5.

Table 5. Beta coefficient

Company	STZ	COXE	SCTO	SNO	CCEV	BATV	ERCA	TLV
Beta	0,75	0,87	1,55	0,60	0,45	0,70	0,62	1,43

It can be seen that two of the companies analyzed appeared to be more risky than the market (β □ 1), SCTO operating in hydraulic structures and TLV in banking field.
The least risky asset relative to the market is Comcereal Vaslui with a coefficient of 0,45.
Market risk-free rate (RFR) was calculated according to the rates of government bonds and have the following values:

RFR_{2007} = 7,277 %; RFR_{2008} = 7,294 % ; RFR_{2009} = 7,459 %; RFR_{2010} = 7,5908 %;
RFR_{2011} = 7,759 %; RFR_{2012} = 7,743 %.

To establish *Security Market Line* was considered the market risk free rate as the arithmetic mean of the interval 2007-2010, 7,4052%.

Volume 6, No. 3, pp. 35–40, © 2014 Pro Universitaria

Market average rate of return was calculated from the average annual return on BET during periods 2007-2010, resulting the following value:

Rm = 10,8775%.

Numerical simulation on SML
Holding period yield: HPY

Suppose that prices will have an ascendent trend:

$$HPY_o = \frac{E_{price_o} - C_{price} + CF}{C_{price}}$$

where E_{price_o} represents the expected price in optimistic scenario; C_{price} is the price at the end of March, considered current price; CF are dividends.
Suppose that prices will have a descendent trend:

$$HPY_p = \frac{E_{price_p} - C_{price} + CF}{C_{price}}$$

where E_{price_o} represents the expected price in pessimistic.

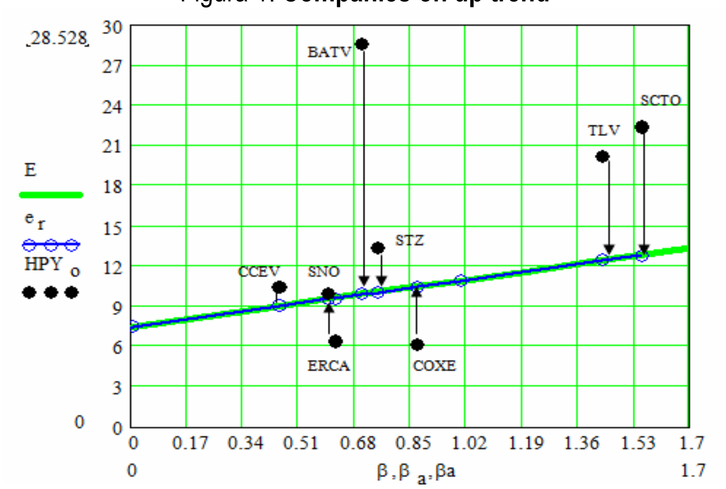
Table 6. Stocks yields

Company	Current price, March 2010	Expected price (moderate optimistic)	Expected price (very pessimistic)	Dividend	Yield (moderate optimistic) HPY_o [%]	Yield (very pessimistic) HPY_p [%]
STZ	0,3	0,34	0,267	0	13,333	-8,0
COXE	99,9	106,0	96,0	0	6,106	-3,904
SCTO	55,0	67,3	52,3	0	22,364	-4,909
SNO	6,050	6,5	5,65	0,15	9,917	-4,132
CCEV	0,125	0,138	0,11	0	10,400	-12
BATV	49,6	60,0	44,8	3,75	28,528	-2,117
ERCA	0,55	0,585	0,52	0	6,364	-5,455
TLV	1,97	2,35	1,850	0,0167	20,137	-5,244

In Figure 1 can view the position of the companies in relation to the SML, based on the above assumptions required.
You can observe that the returns higher than market are found at companies SCTO and LV, and also at the BATV which has the highest return, 28,5%.

In the pessimistic scenario, all companies have much lower returns than those on the SML. Based on observations of prices quotations, on a range outside of the analysis, it can be said that such a negative trend has been reached.

Figura 1. **Companies on up trend**



4. Extension of efficient portfolios

Establishing investment in the previous step allows us to study the behaviour of other securities to extend the number of stocks and get new portfolios.

Performance securities are presented in Table 7.

Table 7. Extension – Return and risk 2007-03.2012

Nr. crt.	**Before crisis 2007-09.2008**		**During the crisis 10.2008-03. 2010**		**Period 2007-03.2010**		**After crisis: 04.2010-2012**	
	ER[%]	***σ[%]***	***ER[%]***	***σ[%]***	***ER[%]***	***σ[%]***	***ER[%]***	***σ[%]***
1□ - **UTBT**	0,68	9,41	3,49	10,60	1,73	10,15	5,862	44,399
2□ - **BMIA**	9,91	43,02	8,40	35,41	9,18	39,51	13,101	60,557
3□ - **SCD**	-1,43	13,89	2,64	14,33	-1,43	13,89	0,145	3,632
4□ - **ATB**	-2,15	8,74	-0,378	21,19	-1,29	16,03	0,071	6,936
5□ - **AMCP**	4,57	29,19	0,939	18,72	2,80	24,72	-0,197	9,798
6□ - **TBM**	-6,30	9,93	0,217	27,83	-3,13	20,93	-0,267	3,666

If we analyze the effects of the crisis on the return of individual assets can be said that:

- Bega Minerale S.A. (BMIA) has a return of 9,18% and 39,51% associated risk. While it is clear the effect of economic crisis, the decline in return is very low, 1,51%;
- Bega Utilaje Constructii S.A. (UTBT) had an increase of return of 2,81% and 1,19% in risk;
- Zentiva S.A. (SCD) had an increase of return of 4,07% and 0,44% in risk;
- Turbomecanica (TBM) had an increase of return of 6,517% and 17.9% in risk.

Of the above companies Bega Mineral SA has been selected to expand portfolio (table 8). Variants selected for development are:

- period 2007-03.2010: 2678, 236;
- during the crisis 10.2008-03.2010: 268.

Tabel 8. Extension - Rentability and risk

Variant	Period	Return %	Risk %	Efficient frontier (%)
2 2 □ 6 7 8	2007-2012	8,7903	26,801	6,8-15,3
2 2□ 3 6	2007-2012	9,8090	34,125	9,2-15,7
2 2□ 6 8	10.2008-03. 2010	6,8000	15,715	6,5-8,79

5. Conclusions

It is noted that the highest return is obtained also for four asset portfolio, but the first variant with the five companies have less risk by about 10% and a return of just one percent lower than the first. Thus, investors are moving without hesitation to portfolio comprising five actions.

References

[1].DIMITRIU,Caracota,M.,CARACOTA,Razvan Constantin (2014) *Investments and Risk*, Ed. dio, Bucureşti, a-2-a editie.

[2]. EDWIN, J., E., MARTIN, J., G. (1995) *Modern Portfolio Theory and Investment Analysis*, John Wiley&Sons, a-5-a ediție, New York.

[3]. HAUGEN, A., R. (1990) *Modern Investment Theory,* Prentice-Hall, a-2 a ediție, New Jersey, Pg 2-3 ; 78-87.

[4]. JAGANNATHAN, R. , WANG, Z. (1998) *An asymptotic theory for estimating beta-pricing models using cross-sectional regression.,* Journal of Finance 53, pg.1285–1309.

[5]. MARKOWITZ, M., H. (2007) *Portfolio Selection*, Blackwell Publishing, a-2-a ediție, Oxford.

[6]. REILLY, K., F., BROWN, C., K. (2006) *Investment Analysis and Portfolio Management,* Thomson South-Western, a-8-a ediție, New York.

[7]. SHARPE, F., W. (2007) *Investors and Markets*, Editura Princeton University Press, Princeton, New Jersey.

1. www.bvb.ro
2. www.startrade.com
3. www.bloomberg.com
4. www.newsin.ro
5. www.sibex.ro
6. www.lesechos.fr